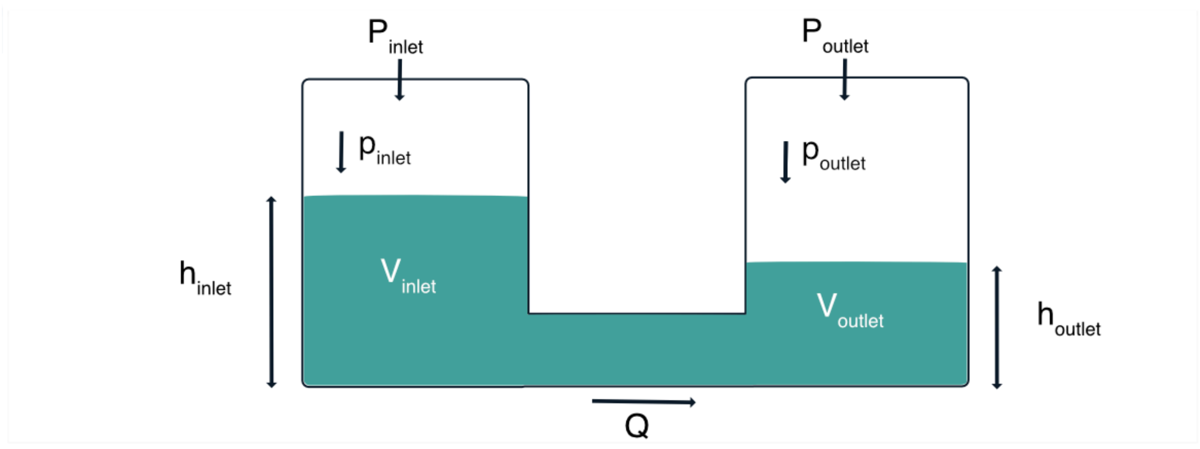
Hydrostatic and Pneumatic Pressure



Our atmosphere applies pressure on all surfaces, liquids can also do so. In case of the atmosphere the weight of the entire gas above is pushing downwards due to gravity creating an isotropic pressure to generate a quite impressive pressure at sea level equivalent to approx. a mass of 1 kg pushing on 1 cm^2. Liquids, however, have a much higher density than air, approx. 1000-fold, and a much more rapid pressure raise with depth is fact. The pressure of a liquid due to its weight is termed *hydrostatic pressure*.

The physical origin can be illustrated by a cylinder (or any other shape) placed on a flat surface *A*. The cylinder with a certain mass and gravitational force *F* will apply a pressure p onto the bottom interface with given area: $p = F/A$. This holds true for solid, liquid and gaseous matter as well. When substituting the force term with the material properties of a liquid, e.g. water, and the geometric properties of the hypothetical cylinder, the hydrostatic pressure will be $p = h \cdot \rho \cdot g$, whereas *h* is the height of the liquid column, ρ (*"rho"*) the liquid density and g the gravitational acceleration. Thus, the hydrostatic pressure does not depend on the size of the cross-sectional area of the liquid column, but only on the height of the liquid column and the density of the liquid. In fact, the hydrostatic pressure is also indifferent to the shape of the hypothetical column.

When working with pressure-driven microfluidic systems, the hydrostatic pressure should always be considered, no matter how small the liquid quantities are. For instance, a one-meter-high water column is already generating a hydrostatic pressure of approx. 10 kPa, or 100 mbars, being quite significant in Microfluidics. Thus, when driving fluids through microfluidic devices, the liquid levels in microfluidic devices can vary considerably in height which is often neglected. Liquid reservoirs of tall and narrow shape appear to be especially prone to systematic pressure deviations. Other common sources of experiment errors are long microfluidic tubings transferring fluids upwards and downwards and even worse if still being partially filled with air. Here, the applied pressure at the source may deviate strongly from the received pressure at the microfluidic device. Once all tubings are filled with liquid and the micro-channel is placed approx. at the same height as the reservoirs and tubings are reduced to minimum, this phenomenon reduces strongly. How to explain this?

Responsible for this is the "*hydrostatic drift*". It can be understood and calculated as follows:

The flow rate Q is proportional to the applied pressure (P) difference plus the hydrostatic pressure (p) difference:

$$Q \sim \Delta P = P_{inlet} - P_{outlet} + p_{inlet} - p_{outlet}$$

The flow resistance R determines the resulting flow rate:

$$Q = \frac{1}{R} \cdot \Delta P$$

Flowing liquid changes the liquid levels in the outlet-reservoir with liquid volume $V_{outlet}(t)$:

$$\frac{dV_{outlet}(t)}{dt} = Q(t)$$

With constant reservoir cross section A, note that $V_{inlet}(t) + V_{outlet}(t)$, $h_{inlet}(t) + h_{outlet}(t)$ and also $p_{inlet}(t) + p_{outlet}(t) = p_{const}$ remain constant, if the total amount of liquid remains unchanged during this experiment.

Hence, the level in the reservoirs changes with volume $V_{oulet}(t) = A \cdot h_{outlet}(t)$ accordingly:

$$\frac{dV_{outlet}(t)}{dt} = A \cdot \frac{dh_{outlet}(t)}{dt} =$$

$$= \frac{1}{R} \cdot \Delta P(t) = \frac{1}{R}(P_{inlet} - P_{outlet}) +$$

$$+ \frac{1}{R}(p_{inlet}(t) - p_{outlet}(t))$$

Hence:

$$\frac{A}{\rho \cdot g} \cdot \frac{dp_{outlet}(t)}{dt} = \frac{1}{R}(P_{inlet} - P_{outlet}) +$$

$$+ \frac{1}{R}(p_{const} - 2 \cdot p_{outlet}(t))$$

which is an inhomogeneous ordinary differential equation of 1st order with an exponential solution rising or decaying with the time scale $\frac{A \cdot R}{2\rho \cdot g}$.

We conclude that less tall reservoirs of larger cross section, or higher hydrodynamic flow-channel resistances reduce the hydrostatic drift.

If the flow rate should remain constant over a long period of time, hydrostatic drift compensation is recommended: The applied pressures have to be conducted such that the right side of the differential equation remains constant. The P²CS has a build-in function to accomplish this compensation automatically, with the function-setup:

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set:hystatic_pressure

set:hystatic_pressure:timescale
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On the other hand, the hydrostatic pressure can also be useful as an excellent source of (quasi-)static pressure for experimental setups. As illustrated, a one-meter large water column can generate significant pressures at the lower end. When doing experiments which require constant pressures with minimal liquid transfer (such that the liquid level of the column will not significantly decrease), water columns present an excellent and inexpensive tool for microfluidic and mesofluidic applications.

Additional information:

www.biophysical-tools.de

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